

Close Tue: 10.3

Close Thu: 14.1, 14.3 (part 1)

Exam 1 will be returned Tuesday.

Read posted solutions, review exam, do not email about grading.

10.3 Polar Coordinates

Polar

Given (r, θ)

1. Stand at origin facing the positive x -axis.

2. Rotate by θ .

pos. = ccw,

neg. = clockwise

3. Walk r -units in direction you are facing.

pos. = forward

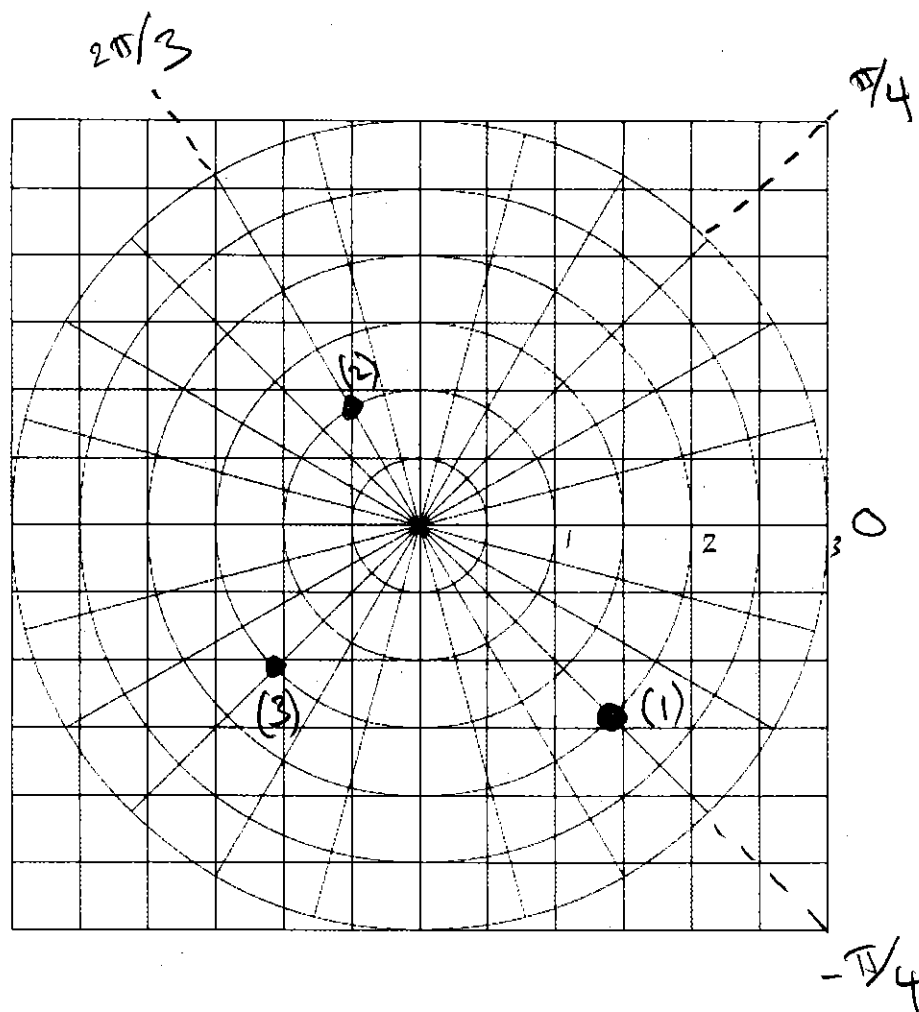
neg. = backward

Entry Task: Plot these polar points

(1) $(r, \theta) = (2, -\pi/4)$

(2) $(r, \theta) = (1, 2\pi/3)$

(3) $(r, \theta) = (-1, \pi/4)$



From trig we already know:

$$x = r \cos(\theta), \quad y = r \sin(\theta)$$

$$\tan(\theta) = \frac{y}{x}, \quad x^2 + y^2 = r^2$$

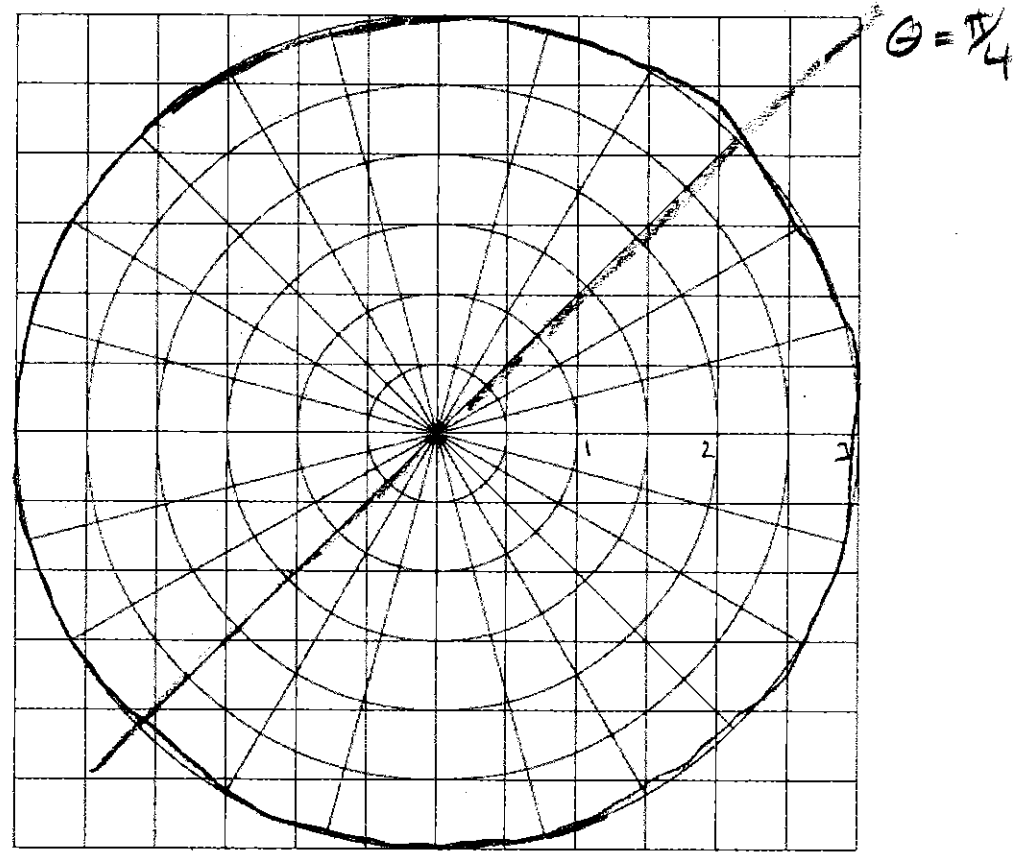
Exercise:

1. Describe all pts where $r = 3$.

$$\Rightarrow x = 3 \cos \theta \quad \text{AND} \quad x^2 + y^2 = 9$$
$$y = 3 \sin \theta \quad \text{CIRCLE!}$$

2. Describe all pts where $\theta = \pi/4$.

$$\theta = \pi/4, \quad r = \text{ANYTHING} \quad \leftarrow \text{SAME}$$
$$\tan(\pi/4) = \frac{y}{x} \quad \leftarrow \text{SAME}$$
$$\Rightarrow y = x$$



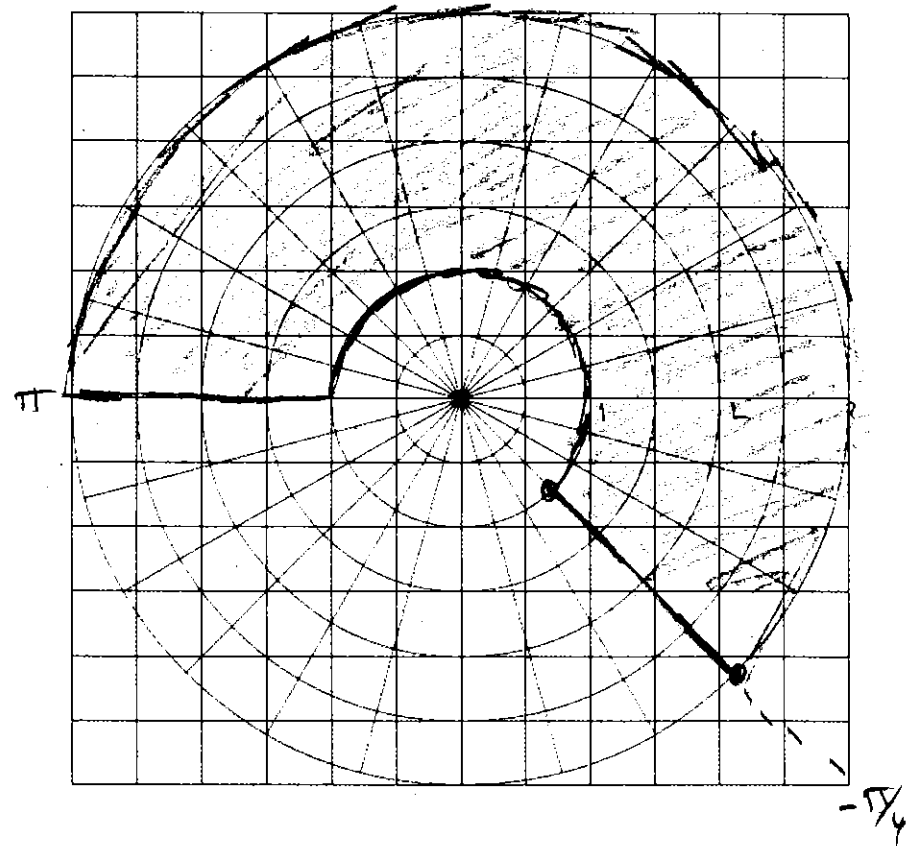
$$r = 3 \quad \theta = \text{ANYTHING} \quad \leftarrow \text{SAME}$$
$$x^2 + y^2 = 9 \quad \leftarrow \text{SAME}$$

Polar Regions

1. Describe all pts where

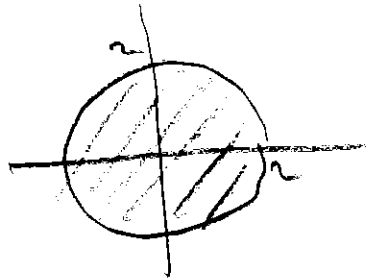
$$-\frac{\pi}{4} \leq \theta \leq \pi \text{ and } 1 \leq r \leq 3$$

↑ "INNER" ↑ "OUTER"



2. Describe all pts where

$$0 \leq \theta \leq 2\pi \text{ and } 0 \leq r \leq 2$$



Plotting Polar Curves

Option 1: Try to convert to x and y .
Then hope you recognize the curve.

Option 2: **Plot points!**

Start with $0, \pi/2, \pi, 3\pi/2$ (intercepts).
For more detail do multiples of $\pi/6$
and $\pi/4$.

Example: Graph $r = \sin(\theta)$

$$r \sin \theta = y \Rightarrow \sin \theta = \frac{y}{r}$$

$$\Rightarrow r = \frac{y}{\sin \theta}$$

$$\Rightarrow r^2 = y$$

$$\Rightarrow x^2 + y^2 = y$$

$$\Rightarrow x^2 + y^2 - y = 0 \quad ??$$

COMPLETE SQUARE

$$x^2 + y^2 - y + \frac{1}{4} - \frac{1}{4} = 0$$

↳ HALF SQUARE

$$x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$$

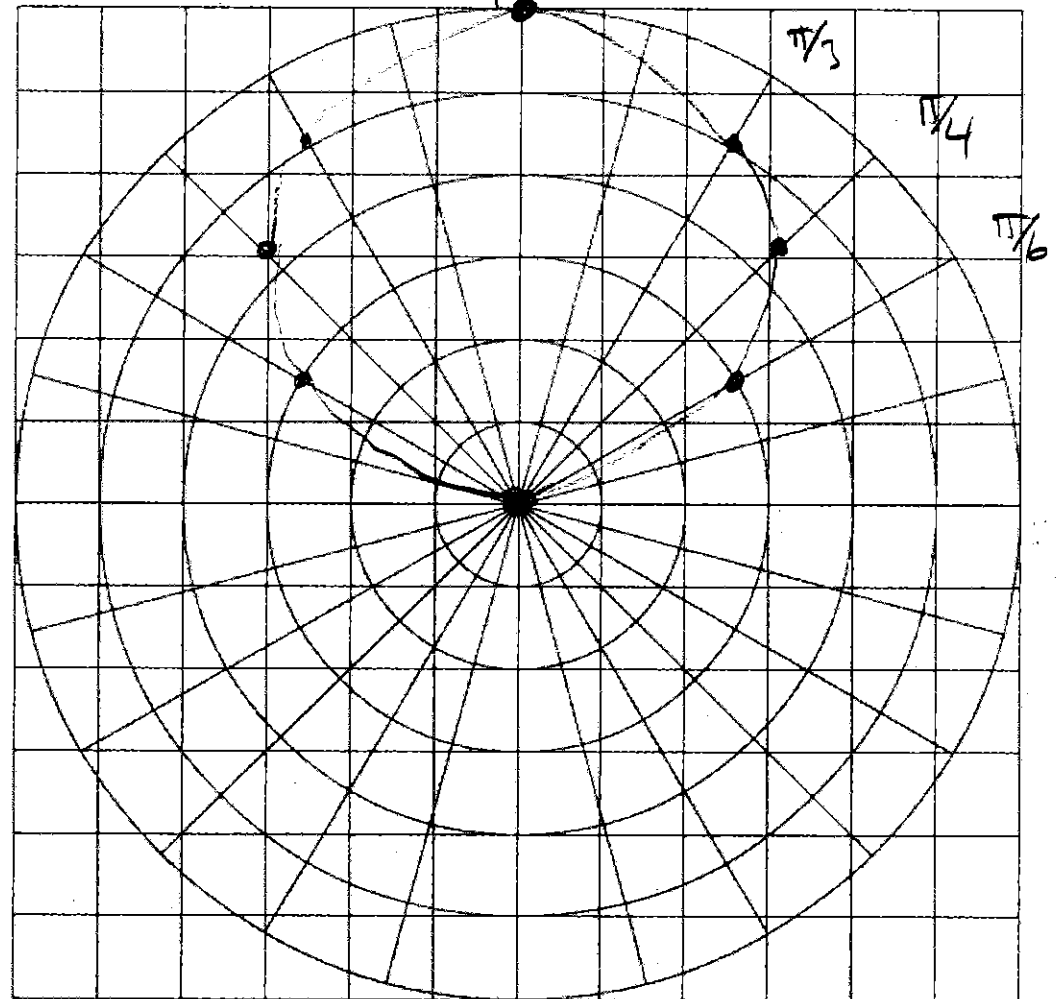
CENTER: $(0, \frac{1}{2})$

RADIUS: $\frac{1}{2}$

θ	0	$\pi/2$	π	$3\pi/2$	2π
r	0	1	0	-1	0

θ	$\pi/6$	$\pi/4$	$\pi/3$	$2\pi/3$	$3\pi/4$	$5\pi/6$
r	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$3/2$	$\sqrt{2}/2$	

≈ 0.7 ≈ 0.86 ≈ 0.86

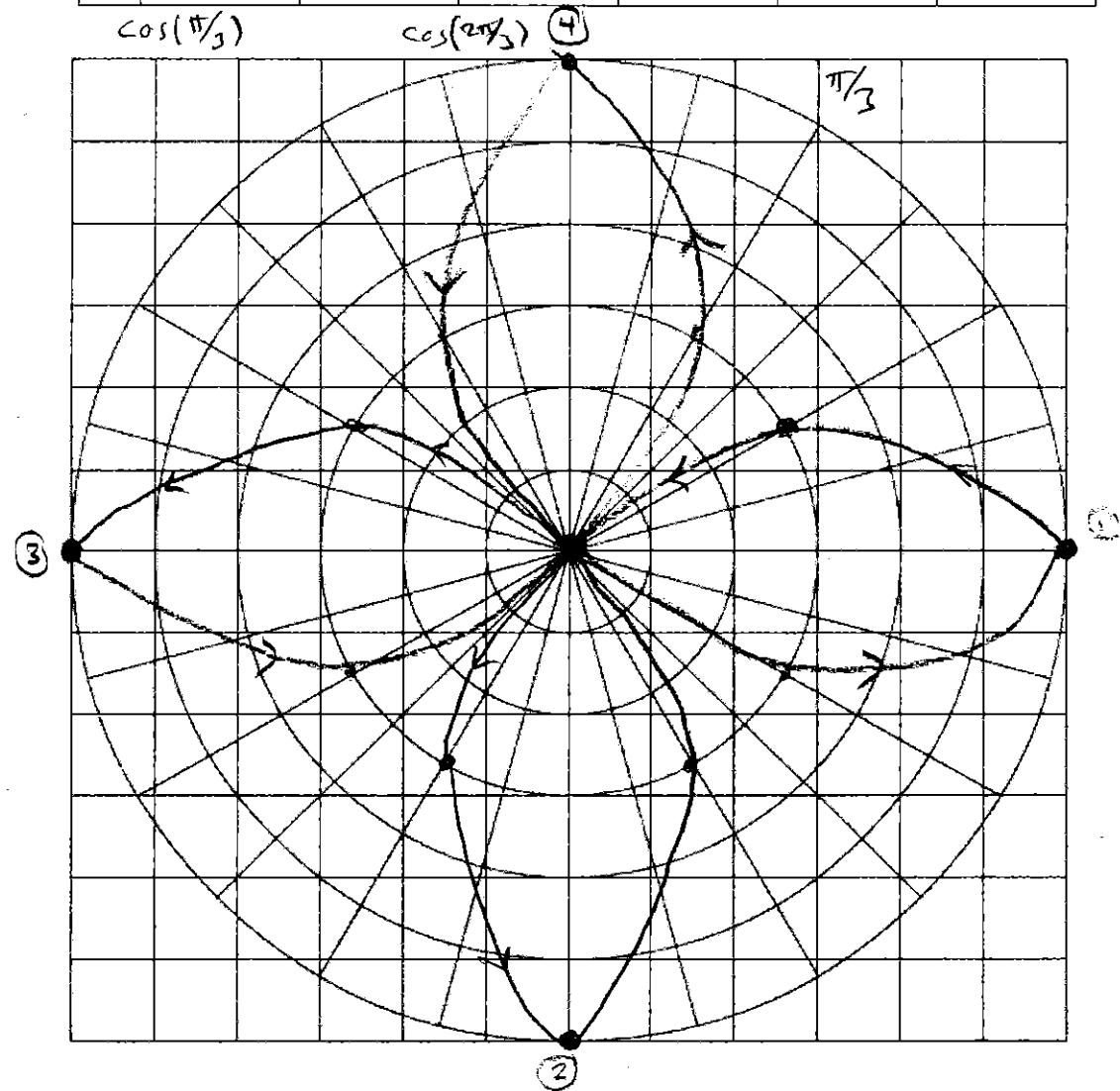


Example: Graph $r = \cos(2\theta)$

θ	0	$\pi/2$	π	$3\pi/2$	2π
r	1	-1	1	-1	1

COULD USE HALF-ANGLE
 THEN TRY TO GET XY-EQUATIONS!
 BUT IT WON'T BE NICE AND
 YOU WON'T RECOGNIZE
 THE XY-CURVE!
 SO JUST PLOT PTS

θ	$\pi/6$	$\pi/4$	$\pi/3$	$2\pi/3$	$3\pi/4$	$5\pi/6$
r	$1/2$	0	$-1/2$	$-1/2$	0	$1/2$



Q DESCRIBE ONE LOOP

$$-\pi/4 \leq \theta \leq \pi/4$$

$$0 \leq r \leq \cos(2\theta)$$

An old exam question:

The four polar equations below each match up with one of the six pictures. Identify which match.

1. $r = \sqrt{\theta}$ ← SPINAL (r getting bigger)
2. $r = 1 - 2\cos(\theta)$
3. $r = 1 + \sin(2\theta)$
4. $r = 9\cos(\theta)$

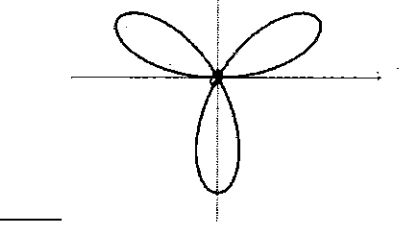
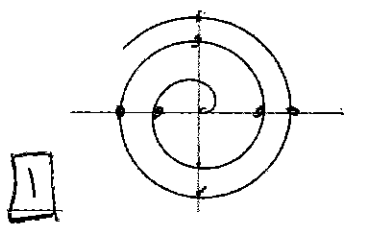
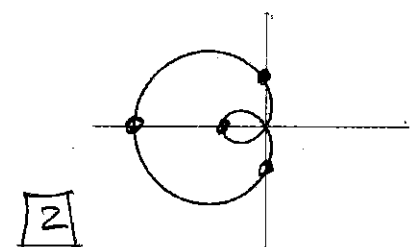
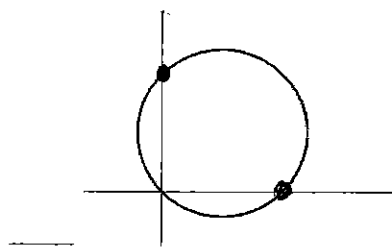
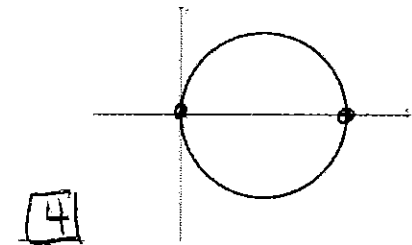
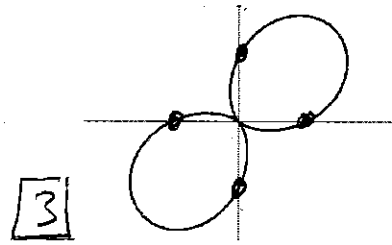
2

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
	-1	1	3	1



3

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
	1	1	1	1



4

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
	9	0	-9	0

